# The Solar Chimney – would a venturi multiply efficiency?

## Summary

In a 1978 patent titled “Utilization of Solar Energy”, Robert E. Lucier suggests that inclusion of a venturi at the base of the solar chimney “increases the air velocity” and that “the impeller of the electrical generator is located within this area.”

In this paper the author attempts to develop this idea theoretically and make quantitative predictions for a solar chimney with a venturi included at the base of the chimney. Calculations and cost estimates on two possible prototypes suggest such a configuration would have good efficiency, be commercially attractive and may allow smaller scale and much shorter height solar chimneys.

The author asks readers with an expertise in computational flow dynamics (CFD) to test these predictions and if successful to publish such results and to make progress towards construction of a prototype solar chimney plus venturi.

## Introduction

The solar chimney concept developed by Schlaich [1, 2] remains a theoretical enigma with the Manzanares prototype built in the 1980’s its single extremely impressive practical realisation. There have been several well developed proposals since that time to build a large scale commercial solar chimney but the height required of up to 1000 metres is quite daunting. Nevertheless there are current plans to build at Ciduad Real in Spain, in Inner Mongolia, China and in Arizona, USA. Several workers have suggested enhancements to reduce costs e.g. Green Tower [3], floating solar chimney [4], inclined solar chimney [5] and perhaps most ingeniously, the use of a man-made mountain hollow [6].

The present author has taken great interest in the above and studied the many research papers and several patents on the solar chimney. This paper sets out to revive and develop a suggestion made by Robert E. Lucier (1978) in a patent on the “Utilization of Solar Energy” [7]. This describes essentially a solar chimney but with a venturi at the base of the chimney. On careful reading of the text describing Figure 2 in the patent, Lucier says that, “The reduced area at the joint of the cones 11 and 12 increases the air velocity, consequently the impeller 8 of the electrical generator 7 is located within this area.” On careful inspection of Lucier’s Figure 2 the diameter of the throat of the venturi is one third of the diameter of the chimney. This would imply that the velocity of air flow through the venturi would be 9 times that through the chimney. The available kinetic energy for a turbine sited in the throat of the venturi is multiplied by a factor of 81.

Lucier does not attempt to make this latter point with any force in his presentation. Sadly there is barely any reference to Lucier’s patents in the solar chimney literature and certainly no development of this suggestion of incorporating a venturi. There are several other patents on the solar chimney that also suggest use of a venturi to multiply air flow velocity e.g. [8, 9, 10, 11] but none has been developed theoretically to make quantitative predictions and certainly none has proceeded to suggest dimensions for a prototype. The present author seeks to remedy this deficit using the suggestions embedded in Lucier's patent.

## Theoretical Development

Consider the solar chimney as represented in Figures 1 and 2 with a venturi at the base of the chimney. It is suggested that the venturi have the form of a convergent-divergent nozzle and that the turbine should be sited in the narrowest region, the throat of the venturi.

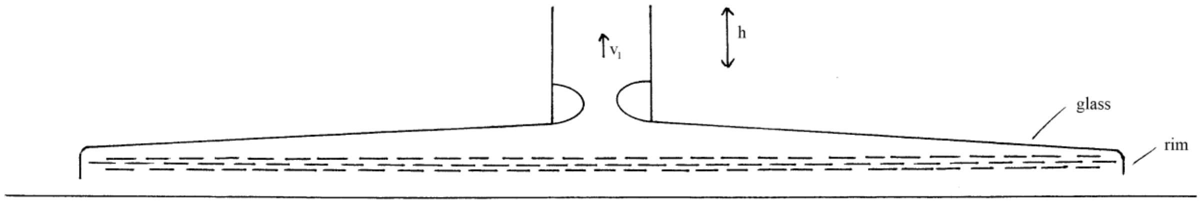

Figure 1

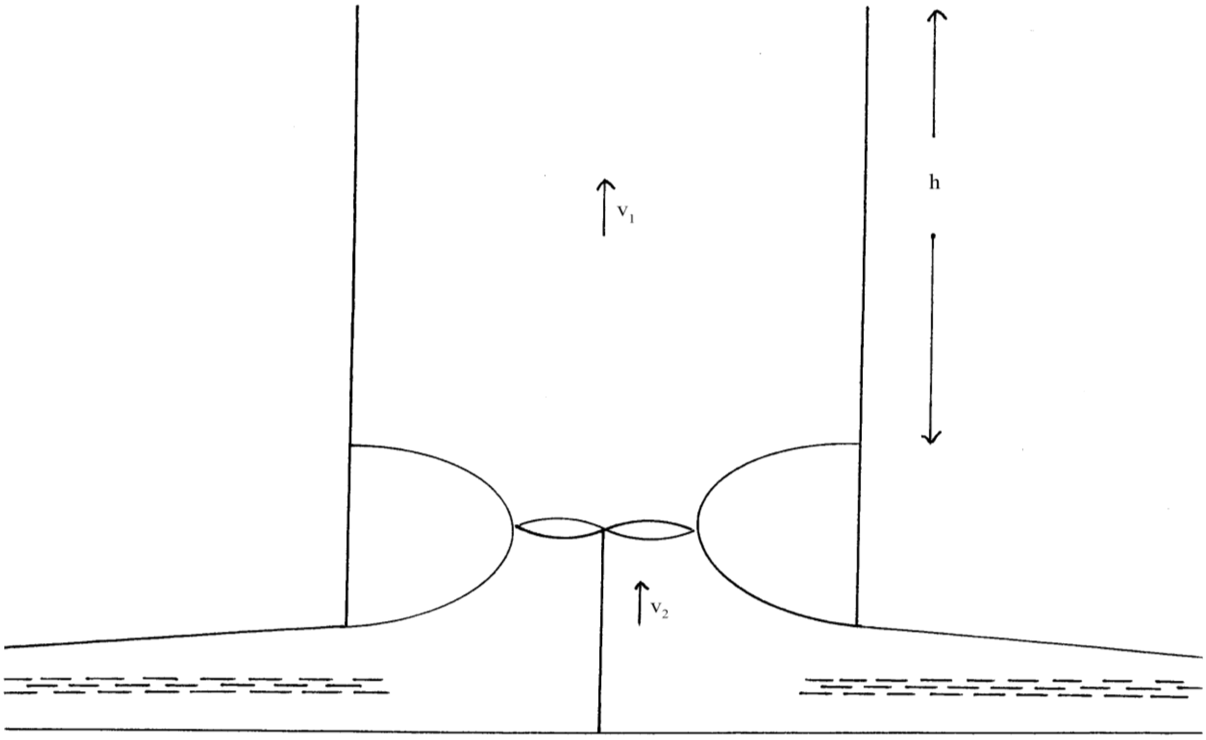

Figure 2 venturi and turbine (enlarged)

Consider

| | |
|---|---|
| $h$ | height of chimney above the venturi |
| $v_1$ | velocity of the air flow through the chimney |
| $A_1$ | cross-sectional area of the chimney |
| $T$ | ambient temperature |
| $\Delta T$ | excess temperature (above ambient) of air in the solar chimney |
| $v_2$ | velocity of air flow through throat of venturi |
| $A_2$ | cross-sectional area throat of venturi |
| $A_3$ | area solar absorber |
| $I$ | insolation |
| $g$ | gravitational constant |
| $\rho$ | density of air at temperature T |
| $C_p$ | heat capacity of air at constant pressure |

Incident solar energy is taken up by the absorber which warms air in its vicinity which rises. At constant insolation, a steady state is established with air temperature throughout the chimney at $T + \Delta T$ and air flow velocity $v_1$.

As the rising air passes from the collector through the venturi it must multiply its velocity through the throat of the venturi. Some of its thermal energy is converted into flow kinetic energy but its temperature falls as a consequence. If there is no turbine present, the air flow will decelerate as it flows through the divergent part of the venturi, its flow kinetic energy is dissipated by turbulence and its temperature recovers to $T + \Delta T$.

When a turbine is interposed in the throat of the venturi it absorbs part or most of the flow kinetic energy exporting this as electricity. Consider that the turbine exports an amount of energy equivalent to $\Delta T'$ in the air flow temperature.

It is the buoyancy of the warm air in the height of the chimney above the venturi that draws air through the configuration. The velocity of air flow in the chimney is given by

$$v_1^{\,2} = 2\,\frac{\Delta T}{T}\,g\,h \tag{1}$$

Constant mass flow requires that

$$\text{flow rate through chimney} = \text{flow rate through throat of venturi}$$

$$v_1 A_1 = v_2 A_2 \tag{2}$$

As air flow is accelerated through the venturi, it suffers a fall in temperature $\Delta T'$ which is the kinetic energy made available to the turbine

$$\tfrac{1}{2} \text{ x mass flow x } v_2^{\,2} = \text{mass flow x } C_p \text{ x } \Delta T'$$

$$v_2^{\,2} = 2\,C_p\,\Delta T' \tag{3}$$

The solar energy absorbed in the collector provides air flow through the chimney at $T + \Delta T$ and energy equivalent to $\Delta T'$ available for export by the turbine

$$\text{total solar energy absorbed} = \text{mass flow x specific heat x temperature rise}$$

$$I\,A_3 = A_1\,v_1\,\rho\,C_p\,(\Delta T + \Delta T') \tag{4}$$

When we consider collectively equations (1) to (4) the values of T I g $\rho$ and $C_p$ are known. It leaves the variables h $v_1$ $v_2$ $A_1$ $A_2$ $A_3$ $\Delta T$ and $\Delta T'$. There are eight variables. If four of these are fixed, the other four can be calculated. In this way an infinite number of solar chimneys could be devised and all their variables calculated. The author will present two such examples.

Please note also that the author believes that rather than place the absorber at ground level, there should be a rim around the perimeter of the solar collector and the solar absorber should be situated above the level of the base of the rim (see Figure 1). This will remove any possibility of back flow and guarantee that all the solar energy taken up by the absorber is transferred to air flow through the collector, venturi and chimney.

It is also the author's belief that the absorber should be multi layered, of up to one metre height and abundantly perforated to allow easy through flow of air. It could be of metallic honeycomb structure with its upper surface coated with efficient solar absorber paint. It is also recognised that the absorber layer has an important role in energy storage e.g. it could be built around water tubes to help reduce daytime peak production storing energy to meet evening/night demand.

## Prototype Large scale Solar Chimney plus Venturi

In the proposals considered for the Schlaich solar chimney it is suggested that the height could be up to 1000 metres and the solar collector area up to 30 km$^2$.

Consider a solar chimney plus venturi as illustrated in Figures 1, 2 where

$h = 100$ m
$A_3 = 1$ km$^2$
$\Delta T = 9$ C
$\Delta T' = 1$ C
$T = 300$ K
$I = 750$ watts/m$^2$
$g = 9.81$ m/s$^2$
$\rho = 1.18$ kg/m$^3$
$C_p = 1005$ joules/kg K

In suggesting the above values for $\Delta T$ and $\Delta T'$ the solar chimney plus venturi is being designed so that 10% of the solar energy absorbed in the collector is made available as flow kinetic energy for a turbine placed in the throat of the venturi i.e. it is being designed for an overall efficiency of 10%.

The insolation considered of 750 watts/m$^2$ represents maximum summer insolation in the UK.

From equation (1) above

$$v_1^{\,2} = 2 \text{ x } \frac{9}{300} \text{ x } 9.81 \text{ x } 100$$

$$v_1 = 7.672 \text{ m/s}$$

From equation (3) above

$$v_2^{\,2} = 2 \times 1005 \times 1$$

$$v_2 = 44.83 \text{ m/s}$$

From equation (4) above

$$A_1 = \frac{750 \times 1000 \times 1000}{7.672 \times 1.18 \times 1005 \times 10}$$

$$A_1 = 8243.4 \text{ m}^2$$

This gives a chimney diameter of 102.5 m

From equation (2) above

$$A_2 = \frac{7.672 \times 8243.4}{44.83}$$

$$A_2 = 1410.7 \text{ m}^2$$

This gives a throat of venturi and turbine diameter of 42.4 m.

The chimney is of height 100 m above the venturi and diameter 102.5 m. The turbine has diameter 42.4 m and axle height about 30 m. The maximum velocity of air flow through the turbine is 44.83 m/s (100 mph). The maximum output of the turbine is 10% of the total insolation

$$0.1 \times 750 \times 1000 \times 1000 \text{ watts}$$

$$75 \text{ MW}$$

If the prototype above was built in the tropics where average annual insolation is 6 kWh/m$^2$/day the average daily output would be

$$0.1 \times 6 \times 1000 \times 1000$$

$$600{,}000 \text{ kWh}$$

## Cost Estimate

The solar chimney now being planned in Arizona, USA is of height 800 m, chimney diameter 300 ft (91.5 m) and of total cost $750 – 1000 million [12]. The chimney itself will cost up to $250 million. The chimney considered above is of height 100 m, diameter 102.5 m and estimated cost

$$\frac{100}{800} \times \frac{102.5}{91.5} \times \$250 \text{ million} = \$35 \text{ million}$$

A turbine of maximum output 75 MW should cost under $75 million. For the solar collector, Zhou et al. [6], page 853, Table 2, quote $113.4 million for the cost of 19.3 km$^2$ collector. These figures give

| | |
|---|---|
| solar chimney | $ 35 million |
| turbine | 75 |
| collector 1 km$^2$ | 6 |
| | $116 million |

The venturi section for the configuration has

| | |
|---|---|
| diameter mouth of venturi | 102.5 m |
| diameter throat of venturi | 42.4 m |
| height estimate | 50 m |

There is no guide in the literature to provide a reliable cost estimate for the venturi but a reasonable speculation would be

| | |
|---|---|
| total cost of solar chimney plus venturi prototype | $150 million |

From above the average annualised electricity production in a tropical climate is 600,000 kWh/day. If this is valued at $0.10/kWh

| | |
|---|---|
| annual value of electricity production | $60,000 x 365<br>$21.9 million |

Thus the solar chimney plus venturi proposed has a repayment period of about 7 years.

## Fluid Mechanics Check

It is possible to double check the accuracy of the theoretical calculations above. Fluid mechanics requires that for air flow from the mouth to the throat of the venturi

$$\frac{T_m}{T_t} = \left[\frac{P_m}{P_t}\right]^{(k-1)/k} = \left[\frac{\rho_m}{\rho_t}\right]^{k-1}$$

where T P and ρ represent respectively the temperature, static pressure and density of air at the mouth (subscript m) and throat (subscript t) of the venturi. The constant k is the ratio of heat capacity at constant pressure to constant volume and has a value 1.4.

Consider the Table below which is partly completed from earlier results (bold type) and partly from calculations below (italics).

| | ambient air | mouth of venturi | throat of venturi |
|---|---|---|---|
| velocity air flow | **0** | **7.672** | **44.83** |
| dynamic pressure | **0** | *35* | *1186* |
| static pressure | **101,300** | *101,265* | *100,114* |
| temperature | **300** | **310** | **309** |
| density | **1.18** | *1.14154* | *1.13222* |

Atmospheric pressure is 101,300 Pascals. The dynamic pressure is given by

$$\tfrac{1}{2} \text{ x density x (velocity air flow)}^2$$

Thus for air flow at the mouth of the venturi, dynamic pressure is

$$\tfrac{1}{2} \text{ x } 1.18 \text{ x } (7.672)^2 = 35 \text{ Pascals}$$

Bernoulli's principle requires that total pressure (static and dynamic) is constant throughout the flow. Thus

$$\text{static pressure at mouth of venturi } P_m = 101{,}300 - 35 = 101{,}265$$

The density of air in the mouth of the venturi is given by the Gas Law

$$\frac{P_a}{T_a\,\rho_a} = \frac{P_m}{T_m\,\rho_m}$$

where the subscript a refers to ambient conditions

$$\rho_m = \frac{P_m}{P_a} \times \frac{T_a}{T_m} \times \rho_a$$

$$= \frac{101{,}265}{101{,}300} \times \frac{300}{310} \times 1.18$$

$$= 1.14154$$

For the throat of the venturi

$$\text{dynamic pressure} = \tfrac{1}{2} \times 1.18 \times (44.83)^2$$

$$= 1186 \text{ Pascals}$$

Thus

$$\text{static pressure in throat of venturi } P_t = 101{,}300 - 1186 = 100{,}114$$

The density of air in the throat of the venturi

$$\rho_t = \frac{100{,}114}{101{,}300} \times \frac{300}{309} \times 1.18$$

$$= 1.13222$$

If we now consider individually the terms in the fluid mechanics equation

$$\frac{T_m}{T_t} = \frac{310}{309} = \underline{1.003236}$$

$$\log \left[\frac{P_m}{P_t}\right]^{(k-1)/k} = \frac{2}{7} \log \frac{101{,}265}{100{,}114}$$

$$= 0.0014184$$

$$\left[\frac{P_m}{P_t}\right]^{(k-1)/k} = \underline{1.003271}$$

$$\log \left[\frac{\rho_m}{\rho_t}\right]^{k-1} = \frac{2}{5} \log \frac{1.14154}{1.13222}$$

$$= 0.0014241$$

$$\left[\frac{\rho_m}{\rho_t}\right]^{k-1} = \underline{1.003285}$$

The values obtained for the three terms in the fluid mechanics equation agree to within 1 part in 20,000. This confirms the accuracy/internal consistency of the theoretical calculations.

## Prototype Small scale Solar Chimney plus Venturi

Using the earlier Figures 1, 2 and theoretical development, consider a small scale solar chimney plus venturi where

$h$ = 20 m
$A_3$ = 10,000 $m^2$
$\Delta T$ = 3 C
$\Delta T'$ = 3 C

$T$ = 300 K
$I$ = 750 watts/$m^2$
$g$ = 9.81 $m/s^2$
$\rho$ = 1.18 $kg/m^3$
$C_p$ = 1005 joules/kg K

In suggesting $\Delta T = \Delta T' = 3$ the solar chimney plus venturi is being designed for an overall efficiency of 50%.

From equation (1) $$v_1^{\ 2} = 2 \times \frac{3}{300} \times 9.81 \times 20$$

$$v_1 = 1.981 \text{ m/s}$$

From equation (3) $$v_2^{\ 2} = 2 \times 1005 \times 3$$

$$v_2 = 77.65 \text{ m/s}$$

From equation (4) $$A_1 = \frac{750 \times 10{,}000}{1.981 \times 1.18 \times 1005 \times 6}$$

$$= 532.1 \text{ m}^2$$

This gives a chimney diameter of 26.0 m.

From equation (2) $$A_2 = \frac{1.981 \times 532.1}{77.65}$$

$$A_2 = 13.57 \text{ m}^2$$

This gives a throat of venturi/turbine diameter of 4.16 m.

## Fluid Mechanics check

If we follow exactly parallel steps to the calculation elaborated earlier, the results are as in the Table below:

| | ambient air | mouth of venturi | throat of venturi |
|---|---|---|---|
| velocity air flow | **0** | **1.981** | **77.65** |
| dynamic pressure | **0** | *2.32* | *3,558* |
| static pressure | **101,300** | *101,298* | *97,742* |
| temperature | **300** | **306** | **303** |
| density | **1.18** | *1.15684* | *1.12728* |

From the fluid mechanics equation

$$\frac{T_m}{T_t} = \left[\frac{P_m}{P_t}\right]^{(k-1)/k} = \left[\frac{\rho_m}{\rho_t}\right]^{k-1}$$

$$\frac{T_m}{T_t} = \frac{306}{303} = 1.009901$$

$$\left[\frac{P_m}{P_t}\right]^{(k-1)/k} = \left[\frac{101{,}298}{97{,}742}\right]^{2/7} = 1.010262$$

$$\left[\frac{\rho_m}{\rho_t}\right]^{k-1} = \left[\frac{1.15684}{1.12728}\right]^{2/5} = 1.010408$$

The above three terms agree to within 1 part in 2000 confirming the accuracy/internal consistency of the earlier calculations.

## Viability of small scale prototype

The turbine required is of diameter 4.16 m, axle height 10-12 m and needs to harness maximum air flow velocity 77.65 m/s (174 mph). The maximum output of the turbine is

$$750 \text{ watts/m}^2 \;\; \text{x} \;\; 10{,}000 \text{ m}^2 \;\; \text{x} \;\; 50\% \text{ efficiency}$$

$$3.75 \text{ MW}$$

If built in a tropical climate with average annual isolation of 6 kWh/m$^2$/day

$$\text{average output} = 0.5 \;\text{x}\; 6 \;\text{x}\; 10{,}000$$

$$= 30{,}000 \text{ kWh/day}$$

Valued at \$0.10/kWh the total value of the electricity produced is \$3,000/day or \$1.1 million/year.

To estimate the cost of construction, consider as follows:

(1) The chimney. The solar chimney now being planned in Arizona, USA is of height 800 m, chimney diameter 91.5 m and of total cost \$750-1000 million [12]. The chimney itself will cost up to \$250 million. In the small prototype under consideration the chimney is of height 20 m and diameter 26.0 m

$$\text{Cost of chimney} \quad = \quad \frac{20}{800} \times \frac{26.0}{91.5} \times \$250 \text{ million} \quad = \quad \$1.78 \text{ million}$$

(2) The turbine. In the literature turbines are costed at less than \$1000/kw. The maximum output above is 3.75 MW giving a turbine of cost of less than \$3.75 million.

(3) The solar collector. Zhou et al. [6], p 853, Table 2 estimates the cost of a solar collector of area 19.3 $km^2$ as \$113.4 million. In this case the solar collector area is 0.01 $km^2$ giving a cost of \$0.06 million.

(4) The venturi. The author has no precedent to base an estimate. The dimensions are mouth of venturi diameter 26.0 m, throat of venturi diameter 4.16 m and total length/ height of about 15 metres. Estimate cost \$3 million.

Thus the total cost of the small scale prototype solar chimney plus venturi is about \$8.6 million giving a repayment period of about 8 years.

## Efficiency of Solar Chimney plus Venturi

In the calculations presented for the larger prototype it is “assumed” that there will be an overall efficiency of 10% whilst the smaller prototype is “designed” for an efficiency of 50%. But what about Carnot? Doesn’t that limit efficiency to $\Delta T/T$?

The author accepts that the Carnot theorem applies to all steam turbines and that it applies widely in thermodynamics. (Note however it does not apply to photovoltaics). If we consider in detail the sequence of energy changes involved in the solar chimney plus venturi, all the individual energy changes are subject only to the First Law of Thermodynamics:

- transmission of solar energy through the glass of the solar collector
- absorption of solar energy by the specially coated solar absorber
- transfer of heat energy from the solar absorber to ambient air in the collector
- convection of heat upwards by ambient air from the solar collector to the venturi
- conversion of thermal energy into flow kinetic energy in the throat of the venturi
- absorption of flow kinetic energy by the turbine and its conversion to electricity
- slow down of air flow post turbine and dissipation of residual kinetic energy as heat
- convection of warm air through the chimney into the atmosphere

Each of these energy changes has high efficiency and not one is subject to the Carnot limit. If each individual change is subject only to the First Law of thermodynamics, then the totality of these energy changes is subject only to the First Law.

There is however a massive energy loss through warm air leaving at the top of the chimney – but it is the buoyancy of this warm air column that drives the system. The amount of this energy loss can be dramatically reduced and the amount of useful energy harnessed massively increased by using a narrow venturi. This physical constriction forces rising warm air from the collector to convert part or most of its excess thermal energy into flow kinetic energy. This can be intercepted by a turbine and exported as electricity. The Carnot theorem has no influence on the dimensions selected for the venturi and can impose no limitation on such a design.

Consider an alternative argument. The buoyancy of air in the chimney at excess temperature $\Delta T$ and with its height $h$ generates a velocity $v_1$ for air flow through the chimney. The geometry of the venturi multiplies this velocity to $v_2$ in the throat of the venturi and massively multiplies available kinetic energy. If $v_2$ is somehow externally constrained by the Carnot theorem, then what happens? If $v_2$ is constrained then it would mean a lower flow rate at higher $\Delta T$ to carry heat from the collector through the chimney. But if $\Delta T$ is higher, $v_1$ is increased giving a still higher $v_2$ .... We are in a riddle. The author believes that $h$ and $\Delta T$ determine $v_1$ and the geometry of the venturi imposes its multiplication to $v_2$. There is no external constraint.

The author is persuaded that the Carnot theorem with its $\Delta T/T$ maximum efficiency cannot apply. There will be significant energy losses through the glass of the solar collector and through the walls of the chimney. There will also be substantial energy loss in the turbine though ducted flow allows turbine efficiency of over 80%. Any energy losses in the turbine will be manifest as heat and will not be lost to the system. They will contribute to the buoyancy of the warm air in the chimney. There is some necessary energy loss in the warm air that provides the buoyancy and that leaves the chimney but this can be dramatically reduced by using a narrow venturi.

It is the author’s belief that the solar chimney plus venturi can be mathematically constructed to a wide range of dimensions with the associated efficiency designed into the system ... of 10% or 50% as in the examples quoted ... and perhaps up to 80%.

## Conclusion

The original idea of inclusion of a venturi at the base of the solar chimney comes from a 1978 patent by Lucier [7]. It would create its own constructional challenges but should be achievable. The author claims that such an addition would multiply air flow velocity and that the kinetic energy available to a turbine placed in the throat of the venturi can be multiplied by a large factor. Such a development could provide the major enhancement needed to make the solar chimney an attractive commercial proposition. It would also allow the construction of smaller scale, shorter height solar chimneys whilst retaining impressive efficiency.

The author has no knowledge of computational flow dynamics (CFD) but asks people that have this expertise to make an assessment of the prototypes suggested in this paper to validate or contradict the theoretical predictions. If CFD studies support the claims made, then the author asks that they be published, that other instructive theoretical studies be pursued and that progress be made towards the construction of a prototype solar chimney plus venturi.

## References

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