<u>The Divergent Solar Chimney</u> – theoretical considerations and possible dimensions for a large-scale prototype

Abstract

There have been several investigations into whether the shape of the chimney might affect the efficiency of the solar chimney but results have been inconclusive. Two recent papers, however, have shown that a divergent solar chimney (DSC) multiplies efficiency. Koonsrisuk and Chitsomboon using computational fluid dynamics (CFD) and a theoretical model found that efficiency could be multiplied "as much as 400 times". Ohya et al using laboratory model solar chimneys found experimentally enhancement by a factor of 2-3-4. The present paper develops its own theoretical model and finds that efficiency is multiplied by $(A_1/A_2)^2$ divergent compared to cylindrical, where A_1 is the area of the top of the chimney and A_2 the area of the base of the chimney. Calculations are presented for a large-scale prototype DSC of height 100m, chimney base diameter 20m and chimney top diameter 100m which give an efficiency of conversion of solar to mechanical energy of 67%.

Introduction

The principles of the solar chimney are well-established but construction of a large-scale prototype is discouraged by the fact that even with a 1000m height chimney efficiency is only about 2%. There have been many suggestions to reduce costs e.g. use of a sloping mountain solar collector, a man-made mountain hollow, a floating solar chimney … Many such enhancement technologies are described in a recent review by Al-Kayiem and Aja [1]. The possibility of using a convergent or divergent chimney has been considered by several workers but results are inconclusive. Recently however there have been two papers published which make outstanding claims for the divergent solar chimney.

Koonsrisuk and Chitsomboon [2] considered six possible chimney and collector shapes (Figure 1). Configuration (a) is chosen to be the reference plant in this study. Its collector has a diameter of 200m and a height of 2m and it has a 100m high chimney with a diameter of 8m. Guided by a theoretical model and using CFD technology the authors find that configuration (f) with a "sloping collector and divergent-top chimney of chimney area ratio of 16 can produce power as much as 400 times that of the reference case." Indeed p405, Table 2 gives an efficiency of conversion of solar to kinetic energy of 64.55%. These are startling results but there has been no further paper from Koonsrisuk and Chitsomboon since 2013 to develop this work and little comment by others in the solar chimney literature.

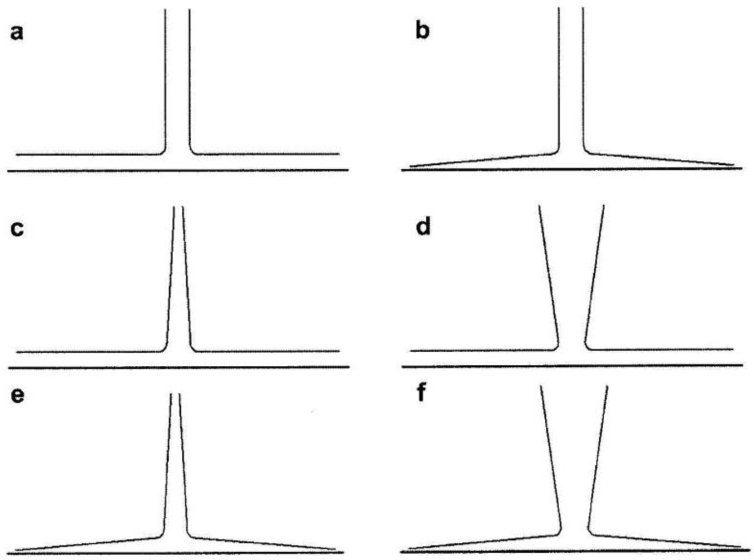

Fig. 2. Schematic layout of (a) reference plant; (b) a sloping collector with a constant-area chimney; (c) a constant-height collector with a convergent-top chimney; (d) a constant-height collector with a divergent-top chimney; (e) a sloping collector with a convergent-top chimney; (f) a sloping collector with a divergent-top chimney.

Figure 1

Ohya et al [3] investigated the use of a diffuser tower in a small model solar chimney using CFD and an experimental approach. The model (Figure 2) was of chimney height 400mm, chimney diameter 60mm and collector diameter 660mm. The divergent chimney was of base diameter 60mm but with a 4-deg open area ratio. The velocity of the air was measured at the base of the chimney. They found that, “The proposed diffuser type tower induces a velocity approximately 1.38-1.44 times greater than the conventional cylindrical type. The wind power generation output is proportional to the cube of the incoming wind velocity into the wind turbine; therefore, approximately 2.6-3.0 times greater output can be expected by using a diffuser type tower.”

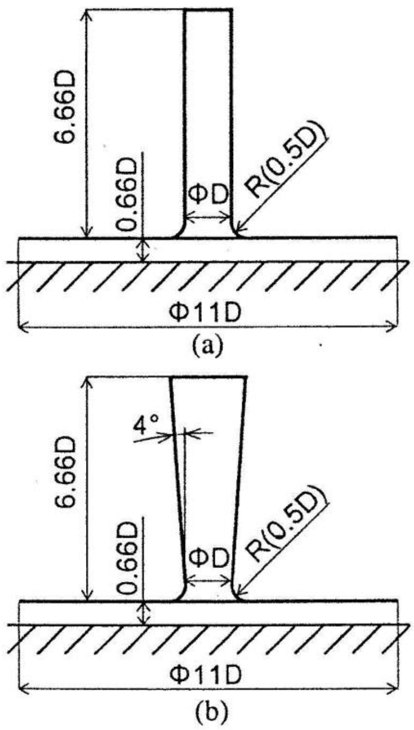

Fig. 5 Dimensions of the experimental model ($D = 60$ mm). (*a*) Cylindrical type tower and (*b*) Diffuser type tower.

Figure 2

This is the first experimental demonstration that shows that a divergent solar chimney can multiply efficiency. In a second study Ohya et al [4] used a larger model of chimney height 2000mm, chimney diameter 320mm and collector diameter 6000mm for the cylindrical type and with an open angle of 8º (4º each side) for the diffuser type. They found that, "The wind velocity for the diffuser tower was greater than that for the cylindrical tower by a factor of 1.5-1.8 times." They conclude that, "The power output obtained in the diffuser tower was 4 times greater than for the cylindrical tower."

These are very significant enhancements – a doubling, trebling or multiplication by 4 of solar chimney output. If the results are scalable to commercial size plants, they could transform the economics of the solar chimney.

Theoretical Considerations – the efficiency of the divergent solar chimney

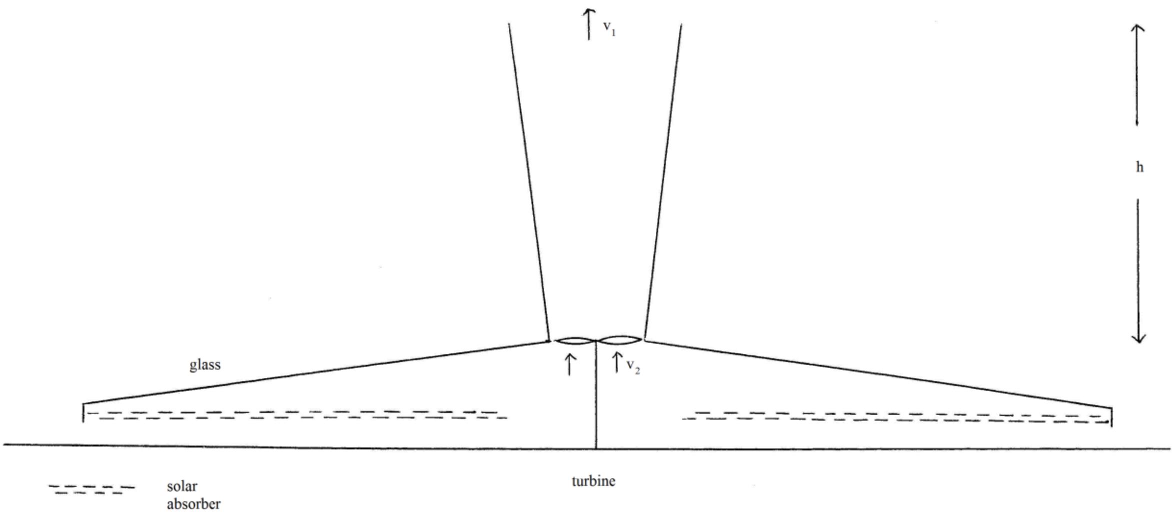

Figure 3

Consider that in Figure 3

h	height of chimney
A_1	cross-sectional area of top of chimney
v_1	velocity airflow through top of chimney
T	ambient temperature
ΔT	excess temperature of airflow as it leaves chimney
A_2	cross-sectional area base of chimney
v_2	velocity airflow at base of chimney
$\Delta T'$	fall in temperature of airflow through turbine
A_3	area solar absorber
I	insolation
g	gravitational constant
ρ	density of air at temperature T
C_p	heat capacity of air at constant pressure and temperature T

Figure 3 represents the divergent solar chimney with a glass roof solar collector. The solar absorber is multi layered with abundant open area to allow easy through flow of rising air. The absorber takes up solar energy with high efficiency warming the air in its neighbourhood which rises because of its lower density. At constant insolation a steady state is established with air flow leaving the chimney with velocity v_1 and at elevated temperature $T + \Delta T$. The velocity of exit air is given by the solar chimney equation [5, 6]

$$v_1^{\,2} \quad = \quad 2 \frac{\Delta T}{T} gh \tag{1}$$

Constant mass flow requires that

mass flow at top of chimney = mass flow at base of chimney

$$A_1 v_1 \rho_1 \quad = \quad A_2 v_2 \rho_2$$

where ρ_1 ρ_2 represent the density of air at the top and base of the chimney respectively. Consider that the temperature change is small and $\rho_1 = \rho_2$ then

$$A_1 v_1 \quad = \quad A_2 v_2 \tag{2}$$

As air flows through the narrow base of the chimney it must acquire a high velocity v_2. The gain in kinetic energy must come from the internal energy of the air whose temperature falls by an amount $\Delta T'$

gain in kinetic energy = mass flow x heat capacity x temperature fall

$$\tfrac{1}{2} \dot{m} v_2^{\,2} \quad = \quad \dot{m} C_p \Delta T'$$

where $\dot{m}$ is the mass flow. Thus

$$v_2^{\,2} \quad = \quad 2 C_p \Delta T' \tag{3}$$

It is assumed that the turbine converts this kinetic energy into electricity with high efficiency.

When we consider air flow through the solar absorber, ambient air at temperature T must be raised in temperature by ΔT which provides the buoyancy of exit air and by an amount $\Delta T'$ which provides the kinetic energy for the turbine

total solar energy absorbed = mass flow x heat capacity x temperature rise

$$I A_3 \quad = \quad \rho A_1 v_1 C_p (\Delta T + \Delta T') \tag{4}$$

$$\text{Efficiency} \quad = \quad \frac{\Delta T'}{\Delta T + \Delta T'}$$

From (3) and (1)

$$\text{Efficiency} \quad = \quad \frac{v_2^{\,2}/2\,C_p}{T\,v_1^{\,2}/2gh \; + \; v_2^{\,2}/2\,C_p}$$

From (2)

$$\text{Efficiency} \quad = \quad \frac{v_2^{\,2}/2\,C_p}{T\,v_2^{\,2}\left(\frac{A_2}{A_1}\right)^2/2gh \; + \; v_2^{\,2}/2\,C_p}$$

Multiply top and bottom by 2 $ghC_p \left(\frac{A_1}{A_2}\right)^2$

$$\text{Efficiency} = \frac{gh\left(\frac{A_1}{A_2}\right)^2}{T\,C_p + gh\left(\frac{A_1}{A_2}\right)^2} \qquad (5)$$

Generally $T\,C_p \gg gh\left(\frac{A_1}{A_2}\right)^2$ and the above reduces to

$$\text{Efficiency} = \frac{gh}{TC_p}\left[\frac{A_1}{A_2}\right]^2$$

When $A_1 = A_2$ the formula is in accurate agreement with Schlaich [6] p118 equation (9) for the cylindrical solar chimney.

But for the divergent solar chimney efficiency is multiplied by $(A_1/A_2)^2$.

This result is in excellent agreement with the concluding arguments of Koonsrisuk and Chitsomboon [2] p 405. It is also in agreement with the formula derived by Padki and Sherif [7] p 347 equation (10) for a convergent solar chimney with the turbine sited at the top of the chimney.

Efficiency of Large-scale Divergent Solar Chimneys

Consider formula (5) above with $h = 100$m $g = 9.81$ ms^{-2} $T = 300$ºK $C_p = 1005$ jkg^{-1}K^{-1} giving $gh = 981$ $TC_p = 301{,}500$. Thus we can derive Table 1

A_1/A_2	$gh\left(\frac{A_1}{A_2}\right)^2$	$TC_p + gh\left(\frac{A_1}{A_2}\right)^2$	Efficiency
1	981	302,500	0.3243%
4	15,700	317,200	4.948
9	79,460	381,000	20.86
16	251,100	552,600	45.44
25	613,100	914,600	67.04
36	1,271,000	1,573,000	80.83

Table 1

Table 1 gives for the cylindrical solar chimney $A_1 = A_2$ of height 100m an efficiency of 0.3243% in accurate agreement with the literature.

For a modestly divergent solar chimney with diameter top of chimney double that of the base of the chimney (A_1/A_2 4) the efficiency is almost 5%.

If we consider more dramatically divergent solar chimneys with top to base diameter ratios of 4, 5 or 6 efficiencies of 45 – 67 – 80% become possible.

The Constructional Challenge of Divergent Solar Chimneys

When we reflect on the geometry of the divergent solar chimney and consider a height of perhaps 100m no one would want to build such a top-heavy unstable structure. To any architect it is a severe challenge. But the entire inverted cone configuration could be contained in a cylindrical framework so that its external appearance is that of a cylindrical solar chimney. The internal structure would hold the cone rigidly in place and be strengthened and shaped around the venturi to withstand the stresses of high acceleration, high velocity air flow. Such elaborate reinforcement would be most needed at the lower levels around the base of the chimney. The upper levels would be undemanding as the air flow is at much lower velocity. The weight of the cylindrical framework would be borne on legs beneath its circumference. The design and construction of a DSC is challenging but achievable.

Possible Dimensions of a Large Scale Prototype

In the earlier section titled "Theoretical Considerations" consider Figure 3, the symbols as defined and equations (1) to (4). For the prototype consider that

h	=	100 m		T	=	300	°K
A_1	=	7850 m^2		g	=	9.81	ms^{-2}
A_2	=	314 m^2		C_p	=	1005	$jkg^{-1}K^{-1}$
A_3	=	4 A_1		ρ	=	1.18	kgm^{-3}
				I	=	750	wm^{-2}

This represents a solar chimney of height 100m, base of chimney diameter 20m, top of chimney diameter 100m giving A_1/A_2 25. The area of the solar absorber is assumed to be 4 times the cross-sectional area of the top of the chimney. Insolation 750 wm^{-2} represents UK summer maximum.

From equation (1)

$$v_1^{\,2} = 2\,\frac{\Delta T}{T}\,gh$$

$$\Delta T = \frac{300\ v_1^{\,2}}{2 \times 9.81 \times 100}$$

$$= 0.1529\ v_1^{\,2}$$

From equation (2)

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{7850}{314} v_1$$

$$v_2 = 25 v_1$$

From equation (3)

$$v_2^{\ 2} = 2 C_p \Delta T'$$

$$\Delta T' = \frac{(25 v_1)^2}{2 \text{ x } 1005}$$

$$= 0.3109 v_1^{\ 2}$$

From equation (4)

$$I A_3 = \rho A_1 v_1 C_p (\Delta T + \Delta T')$$

$$750 \text{ x } 4 A_1 = 1.18 A_1 v_1 \text{ x } 1005 (0.1529 v_1^{\ 2} + 0.3109 v_1^{\ 2})$$

$$v_1^{\ 3} = \frac{750 \text{ x } 4}{1.18 \text{ x } 1005 \text{ x } 0.4638}$$

$$= 5.454$$

$$v_1 = 1.760 \ ms^{-1}$$

$$v_2 = 44.01 \ ms^{-1}$$

$$\Delta T = 0.4736 \ ^oK$$

$$\Delta T' = 0.9634 \ ^oK$$

Thus at insolation 750 wm^{-2} the velocity of air flow through the turbine is 44.01 ms^{-1} (98 mph)

<u>CHECK</u>

Insolation $I A_3 = 750 \text{ x } 4 \text{ x } 7850$

$$= 23.55 \text{ MW}$$

Kinetic energy through turbine $= \frac{1}{2} \rho A_2 v_2^{\ 3}$

$$= \frac{1.18}{2} \text{ x } 314 \text{ x } (44.01)^3$$

$$= 15.79 \text{ MW}$$

Heat loss in exit air = mass flow x heat capacity x temperature rise

$$= \rho A_1 v_1 C_p \Delta T$$

$$= 1.18 \text{ x } 7850 \text{ x } 1.760 \text{ x } 1005 \text{ x } 0.4736$$

$$= 7.760 \text{ MW}$$

This gives an efficiency of $\frac{15.79}{23.55} = 67.05\%$

Figure 4 gives an impression of the prototype divergent solar chimney and is drawn to scale.

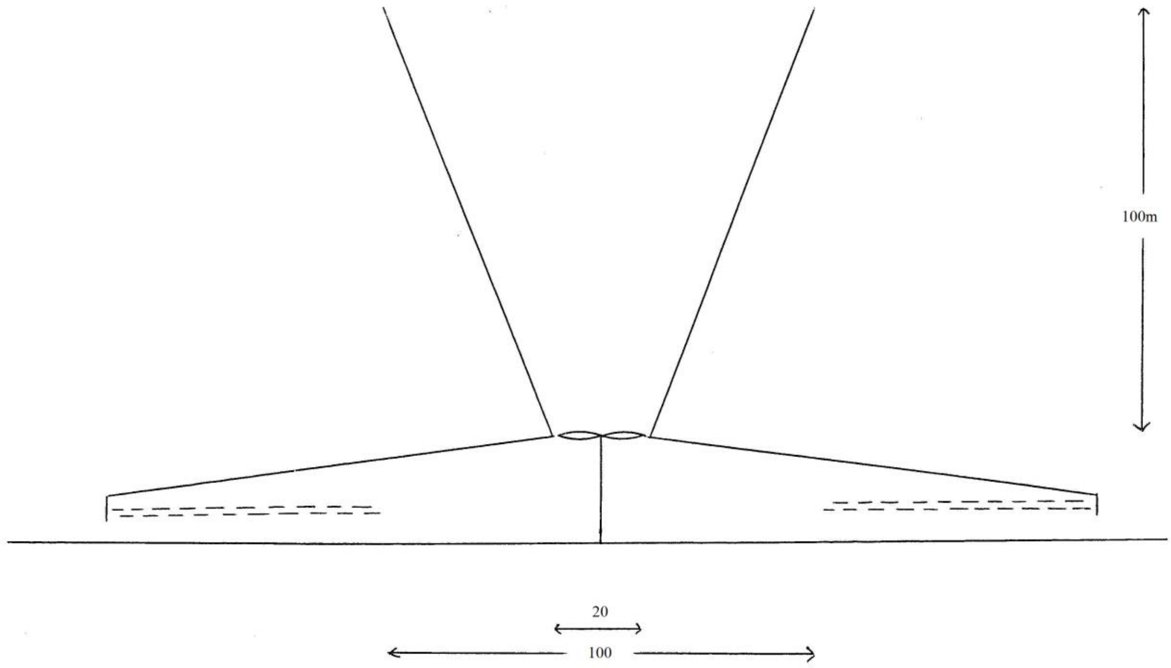

Figure 4

Conclusion

The concept of a divergent solar chimney looks unstable and top-heavy but it could be built with an outer cylindrical support framework. The theoretical model developed in this paper shows that when compared to the cylindrical solar chimney, the efficiency of conversion of solar energy absorbed to the kinetic energy of air flow at the base of the chimney is multiplied by $(A_1/A_2)^2$ in the divergent solar chimney where A_1 is the cross-sectional area of the chimney top and A_2 the area of the chimney base. Calculations are presented for a large-scale prototype which give an efficiency of 67%. The author asks experts on the solar chimney to consider the proposal and to conduct theoretical and experimental work to validate and develop the divergent solar chimney.

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