Proposal for a rectangular solar chimney with multiple turbines in uniform distribution throughout the solar collector

Abstract

A proposal is developed involving a rectangular solar chimney of 100 m height with a rectangular solar collector containing 10 rows of 10 identical turbines symmetrically sited. Incoming air flows through alternate stretches of solar absorber and convergent-divergent nozzle plus turbine. The air flow created by the chimney goes through 10 cycles of heating by the solar absorber then energy removal by a turbine. Calculations show conversion of solar energy into available kinetic energy with very high efficiency.

Introduction

The solar chimney as developed by Schlaich et al. [1] and recently reviewed by Guo et al. [2] offers the possibility of large scale generation of electricity using natural convection. The reasons holding back development concern its height (up to 1000m) and efficiency (about 2%).

The proposal outlined in this paper is based on the solar chimney but has three additional features:

(1) The air flow created by warm air in the chimney is required to pass through a convergent-divergent nozzle or venturi to multiply flow velocity. The turbine is then placed in the throat of the nozzle to harness the much higher flow kinetic energy.
(2) A rectangular configuration is suggested for the chimney and a long, narrow solar collector. This would be easier to build and could allow multiple units to be built in parallel.
(3) The author wondered what would happen if air flow created by the chimney was required to flow through several turbines in series. Would the extra turbines yield greater efficiency?

Linear Configuration with one turbine

Consider Figure 1

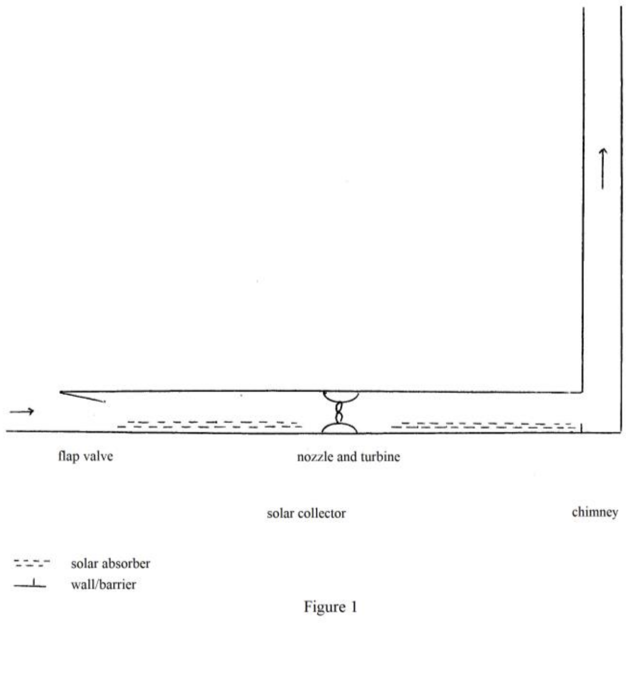

Figure 1

The solar collector has a square cross-section and has a roof and sides made of glass or transparent plastic. The tall chimney also has square cross-section. Solar energy generates warm air which rises into the chimney. The flap valve and low wall/barrier prevent back flow. The nozzle and turbine can be sited anywhere along the length of the collector.

Consider that in Figure 1

h	height from solar absorber to top of chimney
A_1	area cross-section of chimney
A_2	area cross-section of turbine
A_3	area solar absorber
v_1	velocity of air flow through chimney
v_2	velocity of air flow through turbine
T	ambient temperature
ΔT	excess temperature (above ambient) of exit air
$\Delta T'$	fall in temperature as air flows through turbine
g	gravitational constant
ρ	density of air at atmospheric pressure and temperature T
C_p	heat capacity of air at constant pressure and temperature T
I	insolation

The velocity of air flow in the chimney is given by [1]

$$v_1^2 = 2 \frac{\Delta T}{T} gh \qquad (1)$$

Constant air flow requires that

air flow in chimney = air flow through turbine

$$A_1 v_1 = A_2 v_2 \qquad (2)$$

As air flows through the nozzle it is accelerated to velocity v_2. The extra flow kinetic energy comes from the internal energy of the air causing a fall in temperature $\Delta T'$

gain in kinetic energy = mass flow x heat capacity x fall in temperature

$$\tfrac{1}{2} \dot{m} v_2^2 = \dot{m} C_p \Delta T'$$

where $\dot{m}$ is the mass flow

$$v_2^2 = 2 C_p \Delta T' \qquad (3)$$

Solar energy taken up by the absorber provides ΔT for the chimney and $\Delta T'$ for the turbine

total solar energy absorbed = mass flow x heat capacity x temperature rise

$$I A_3 = \rho A_1 v_1 C_p (\Delta T + \Delta T') \qquad (4)$$

Equations (1) – (4) define all the energy changes taking place in Figure 1 and can be used to evaluate different models. Consider that in Figure 1 the turbine is of diameter 3m, the solar collector has cross-section 10 x 10 and length 100m and that the chimney has height 100m and cross-section 10 x 10.

h	=	100	m	T	=	300	^oK
A_1	=	100	m^2	g	=	9.81	ms^{-2}
A_2	=	7.065	m^2	ρ	=	1.18	$kg\ m^{-3}$
A_3	=	1000	m^2	C_p	=	1005	$j\ kg^{-1}\ K^{-1}$
				I	=	750	$w\ m^{-2}$

The insolation considered represents summer UK maximum.

From equation (1)

$$v_1^{\ 2} = \frac{2\ \Delta T}{300} \times 9.81 \times 100$$

$$v_1^{\ 2} = 6.54\ \Delta T \qquad (1)$$

From equation (2) $\quad 100\ v_1 = 7.065\ v_2$

$$v_2 = 14.15\ v_1 \qquad (2)$$

From equation (3) $\quad v_2^{\ 2} = 2010\ \Delta T' \qquad (3)$

From equation (4)

$$750 \times 1000 = 1.18 \times 100 \times 1005\ v_1\ (\Delta T + \Delta T')$$

From (1) (2) and (3) above

$$v_1 \left(\frac{v_1^{\ 2}}{6.54} + \frac{200.3}{2010}\ v_1^{\ 2} \right) = \frac{750{,}000}{118{,}590}$$

$$v_1^{\ 3}\ (2010 + 1310) = 6.324 \times 6.54 \times 2010$$

$$v_1^{\ 3} = 25.04$$

$$v_1 = 2.926$$

From (2) $\quad v_2 = 41.41$

From (1) $\quad \Delta T = 1.309$

From (3) $\quad \Delta T' = 0.8531$

Maximum insolation $\quad I\ A_3 = 750$ kw

maximum kinetic energy $= \frac{1}{2}\ \rho\ A_2\ v_2^{\ 3}$

$$= \frac{1.18}{2} \times 7.065 \times (41.41)^3$$

$$= 296.0 \text{ kw}$$

This gives an efficiency of 39.46 %

Thus a linear configuration with one turbine at maximum insolation gives air flow through the turbine of velocity 41.41 ms^{-1} generating maximum kinetic energy of 296.0 kw with an efficiency of 39.46%.

Linear Configuration with two turbines

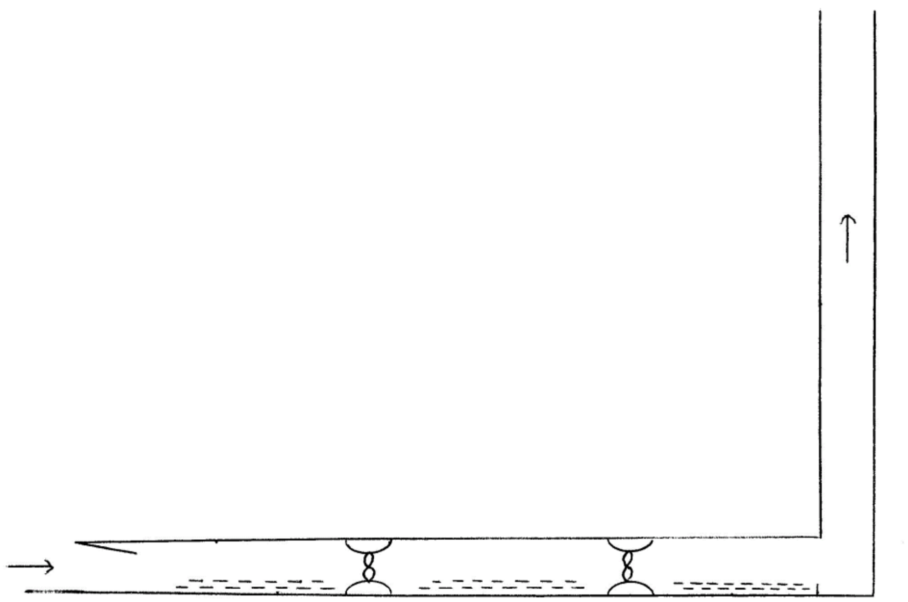

Figure 2

Consider the same configuration and dimensions as in Figure 1 with 2 identical turbines symmetrically placed at about ⅓ and ⅔ the distance along the solar collector (Figure 2). The velocity of air flow through each turbine must be the same, v_2, and there will be an equivalent fall in temperature $\Delta T'$ at each turbine.

From equation (1)

$$v_1^{\,2} = \frac{2\ \Delta T}{300} \times 9.81 \times 100$$

$$v_1^{\,2} = 6.54\ \Delta T \qquad (1)$$

From equation (2)

$$100\ v_1 = 7.065\ v_2$$

$$v_2 = 14.15\ v_1 \qquad (2)$$

From equation (3)

$$v_2^{\,2} = 2010\ \Delta T' \qquad (3)$$

For equation (4) solar energy absorbed must provide ΔT for air flow in the chimney and ΔT′ for each turbine

$$I\,A_3 = \rho\,A_1\,v_1\,C_p\,(\Delta T + 2\,\Delta T')$$

$$750 \times 1000 = 1.18 \times 100 \times 1005\;v_1\,(\Delta T + 2\,\Delta T')$$

From (1) (2) and (3)

$$v_1\left(\frac{v_1^{\,2}}{6.54} + \frac{2 \times 200.3}{2010}\,v_1^{\,2}\right) = \frac{750{,}000}{118{,}590}$$

$$v_1^{\,3}\,(2010 + 2620) = 6.324 \times 6.54 \times 2010$$

$$v_1^{\,3} = 17.95$$

$$v_1 = 2.619$$

From (2) $\quad v_2 = 37.06$

From (1) $\quad \Delta T = 1.048$

From (3) $\quad \Delta T' = 0.6834$

Maximum insolation $\quad I\,A_3 = 750$ kw

maximum kinetic energy per turbine $\quad = \dfrac{1.18}{2} \times 7.065 \times (37.06)^3$

$\quad = 212.2$ kw

For the 2 turbines this gives maximum kinetic energy of 424.4 kw and an overall efficiency of 56.59%.

Thus the result of adding a second turbine is to reduce air flow velocity in the chimney and through the turbines and to reduce exit air temperature. But because there are now 2 turbines harnessing energy the total available kinetic energy is raised from 296.0 kw to 424.4 kw and overall efficiency raised from 39.46 % to 56.59%.

Linear Configuration with 3, 5 or 10 turbines

Similar calculations to the above have been carried out for configurations involving 3, 5 or 10 turbines in series in the solar collector. The results are presented in Table 1. For each case, chimney height $h = 100$m, chimney cross-section $A_1 = 10 \times 10$m, turbine diameter 3m, collector height and width 10m and absorber area $A_3 = 100 \times 10$m. As the number of turbines is increased, the solar collector will need to be of greater length but the area of the solar absorber has been assumed as $A_3 = 1000\text{m}^2$ in each case.

Number of turbines	1	2	3	5	10	
v_1	2.926	2.619	2.410	2.133	1.765	ms^{-1}
v_2	41.41	37.06	34.11	30.20	24.99	ms^{-1}
ΔT	1.309	1.048	0.8879	0.6960	0.4765	°K
$\Delta T'$	0.8531	0.6834	0.5788	0.4537	0.3106	°K
maximum KE per turbine	296.0	212.2	165.4	114.8	65.03	kw
max KE	296.0	424.4	496.2	573.9	650.3	kw
efficiency	39.46	56.59	66.17	76.52	86.70	%

Table 1 Linear configuration with 1 -10 turbines

As we review the Table from left to right, the velocity of air flow through the chimney, v_1, and turbines, v_2, falls as the number of turbines increases. ΔT and $\Delta T'$ are about 1°C or less. Total output increases dramatically as the number of turbines increases and overall efficiency increases from 39.46% to 86.70%. These are very impressive figures – the former is simply due to the venturi effect of requiring air flow to pass through the relatively small cross-section of the turbine. The latter shows that if there is a multiplicity of turbines in series, the velocity of air flow through each turbine is reduced substantially but the total kinetic energy harnessed can be up to 86.70% of the solar energy absorbed.

Why is such high efficiency possible?

The idea of investigating a number of turbines in series comes from a study of steam turbines in power stations. High energy steam flows through a succession of stages with a series of stationary blades and turbine blades and with the steam flow losing energy at each stage. Some commercial turbines have as many as 25 stages [3].

When we consider the solar chimney, it is the air column in the chimney at a temperature of $T + \Delta T$ that provides buoyancy drawing ambient air through the configuration. Whatever work is achieved by the airflow, the heat content that provides the excess temperature ΔT is lost to the system. In the conventional solar chimney it is used ONCE as it flows through the turbine and is then rejected. Why not use the air flow a second, third … n times? It allows the second, third .. turbine to extract some of the residual kinetic/thermal energy from the first turbine … it is a kind of recycling or energy recovery. We have incurred considerable expense in building a tall chimney and large area solar collector, why not use the air flow we have created a second, third … n th time? Indeed the solar collector could have alternate lengths of solar absorber, then nozzle and turbine, then solar absorber, then nozzle and turbine … in many repeat cycles. It is a case of adding and then removing energy in successive cycles to and from an air flow created by the chimney.

In the model above, if there are 10 such cycles, each turbine removes $\Delta T'$ from the air flow and each stretch of solar absorber adds $\Delta T' + 0.1\ \Delta T$ to the temperature of the air flow. The energy loss through the chimney is ΔT but the energy harvested by the turbines is $10\ \Delta T'$, hence the high efficiency.

Solar Absorber/Nozzle and Turbine Cycles

It is suggested that the solar collector comprise regular alternate lengths of solar absorber then nozzle and turbine in repeat pattern as depicted in Figure 3 with generous distances allowed for acceleration of air flow into the convergent nozzle and deceleration in the divergent section. The turbine is sited at the throat of the nozzle.

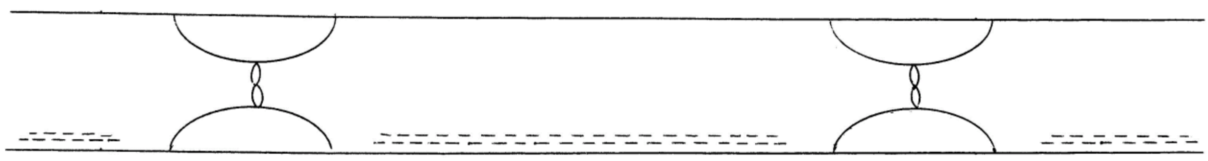

Figure 3

Parallel array of Linear Configurations

The linear configurations hitherto described could be built in parallel allowing some economies of scale. A transverse arrangement (Figure 4) is also possible where incoming air flows through a parallel set of turbines, then through a continuous absorber section, then turbines in repeat cycles. This latter arrangement has the advantage that if one or more of the turbines malfunctioned its workload would be transferred smoothly to the other turbines.

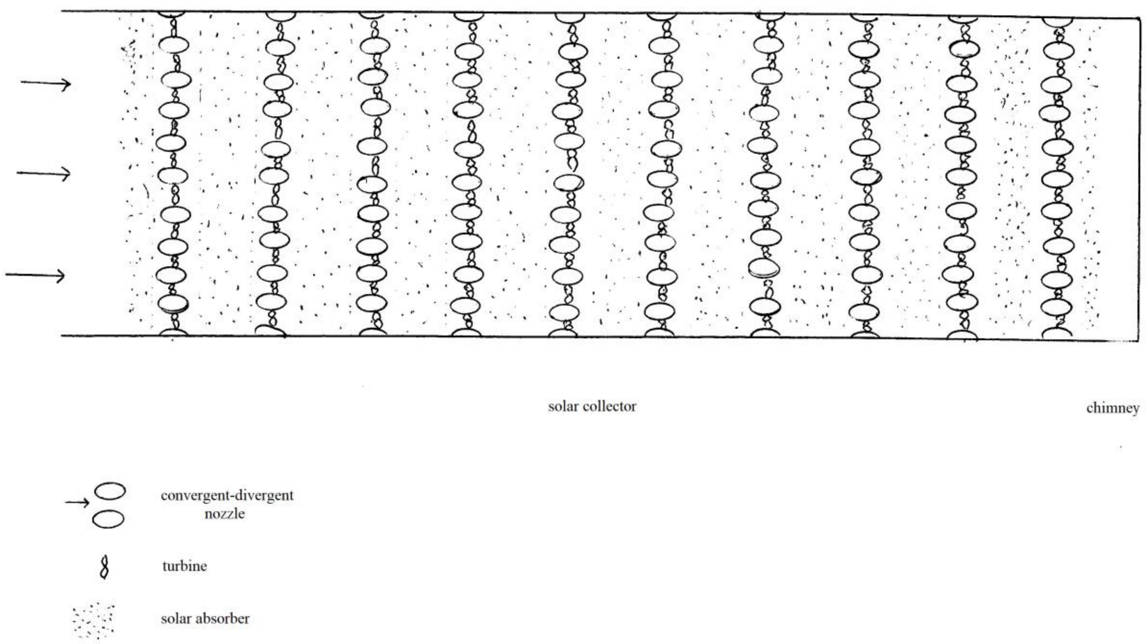

Figure 4

Prototype suggested – a rectangular solar chimney with 10 x 10 turbines in uniform distribution throughout the solar collector

An aerial view of the configuration is presented in Figure 4 and a ground level view in Figure 5.

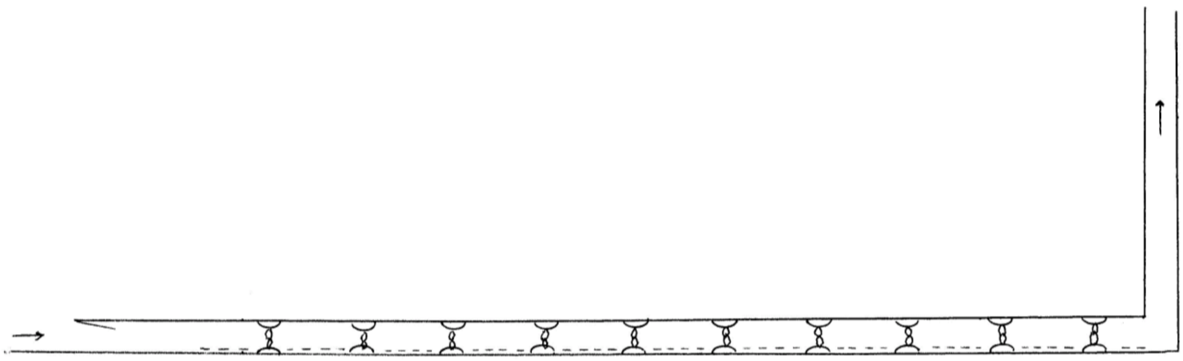

Figure 5

Consider

chimney height	100 m	solar absorber length	18 m
turbine diameter	3 m	nozzle and turbine length	12 m
collector height	10 m	solar collector length	300 m
collector width	100 m	total area solar absorber	
chimney width	10 m	$A_3 = 18 \times 10 \times 10 \times 10$	
chimney length	100 m	$= 18{,}000\ m^2$	

There are 100 identical turbines each of cross-sectional area $A_2 = 7.065\ m^2$

h	=	100	m	T	=	300	°K
A_1	=	1000	m^2	g	=	9.81	ms^{-2}
A_2	=	7.065	m^2	ρ	=	1.18	$kg\ m^{-3}$
A_3	=	18,000	m^2	C_p	=	1005	$j\ kg^{-1}\ K^{-1}$
				I	=	750	$w\ m^{-2}$

From equation (1)

$$v_1^{\ 2} = \frac{2\ \Delta T\ g\ h}{T}$$

$$v_1^{\ 2} = \frac{2\ \Delta T}{300} \times 9.81 \times 100$$

$$v_1^{\ 2} = 6.54\ \Delta T \qquad (1)$$

From equation (2)

rate of air flow through chimney = rate of air flow through row of 10 turbines

$$A_1\ v_1 = 10\ A_2\ v_2$$

$$1000\ v_1 = 70.65\ v_2$$

$$v_2 = 14.15\ v_1 \qquad (2)$$

From equation (3) for each individual turbine

$$v_2^{\ 2} = 2\ C_p\ \Delta T'$$

$$v_2^{\ 2} = 2010\ \Delta T' \qquad (3)$$

For equation (4) the total solar energy absorbed by the configuration must provide the entire air flow with a temperature rise of ΔT for the chimney and ΔT′ for each of the 10 turbines in its flow path.

$$I\ A_3 = \rho\ A_1\ v_1\ C_p\ (\Delta T + 10\ \Delta T')$$

$$750 \times 18{,}000 = 1.18 \times 1000 \times 1005\ v_1\ (\Delta T + 10\ \Delta T')$$

From (1) (2) and (3)

$$v_1 \left(\frac{v_1^{\ 2}}{6.54} + \frac{10 \times 200.3}{2010}\ v_1^{\ 2} \right) = \frac{13.5 \times 10^6}{1.1859 \times 10^6}$$

$$v_1^{\ 3}\ (2010 + 13{,}100) = 11.38 \times 6.54 \times 2010$$

$$v_1^{\ 3} = 9.902$$

$$v_1 = 2.147\ ms^{-1}$$

From (2) $v_2 = 30.39\ ms^{-1}$

From (1) $\Delta T = 0.7051\ °K$

From (3) $\Delta T' = 0.4596\ °K$

Maximum insolation $= I\ A_3$

$= 13.5$ MW

Maximum kinetic energy per turbine $= \frac{1}{2}\ \rho\ A_2\ v_2^{\ 3}$

$$\frac{1.18}{2}\ (7.065)\ (30.39)^3$$

$= 117.0$ kw

For 100 turbines maximum KE $= 11.70$ MW

Efficiency $= 86.70\%$

Thus for the prototype suggested at maximum insolation the velocity of air flow through the turbines is 30.39 ms^{-1} (65 mph). Maximum kinetic energy is 11.70 MW giving an efficiency of 86.70%.

The total area is about 330 x 100 m. If such a configuration was built in repeat patterns it would yield maximum kinetic energy of over 300 MW/km^2.

Additional Comments

- No allowance has been made in the calculations for energy losses e.g. through the solar collector or in the turbines. In the model, ΔT and $\Delta T'$ are both below 1°C so losses through the glass of the collector should be minimal. Energy losses in the turbines will be manifested as heat and will be carried in the air flow into exit air. They will contribute to the buoyancy of the air in the chimney and are effectively recycled.

- Energy storage can be added using water bags placed on the ground beneath the solar absorber [1]. In this way excess daytime heat can be stored and used for evening/night generation.

- The author asks experts on the solar chimney/solar energy to consider the above proposal, to check and challenge the physics used in the calculations, to carry out alternative theoretical assessment and ultimately to build the experimental prototype suggested.

Conclusion

A linear configuration of the solar chimney is described with a rectangular chimney and a long, narrow, rectangular solar collector. The turbine suggested is of small cross-sectional area compared to the cross-sectional area of the chimney. Air flow velocity through the chimney is thus multiplied through the turbine giving an efficiency of conversion of solar into kinetic energy of almost 40%. It is then shown that by having 2, 3, 5 … 10 identical turbines in series in the linear configuration, the overall efficiency can be raised to over 80%. A prototype large configuration is proposed with chimney height 100 m and 10 rows of 10 turbines uniformly distributed in a solar collector area of 100 x 330 m. It is calculated that conversion of solar energy into kinetic energy yields up to 11.70 MW at maximum insolation with an efficiency of 86.70%. The author asks experts on the solar chimney to investigate, assess and develop the proposal.

References

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[3] Fundamentals of Thermodynamics, Adams and Hilding, Harper, 1945, p267-9.

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